Long-memory and the Sea Level-Temperature Relationship: a Fractional Cointegration Approach

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Abstract

Through thermal expansion of oceans and melting of land-based ice, global warming is very likely contributing to the sea level rise observed during the 20th century. The amount by which further increases in global average temperature could affect sea level is only known with large uncertainties due to the limited capacity of physics-based models to predict sea levels from global surface temperatures. Semi-empirical approaches have been implemented to estimate the statistical relationship between these two variables providing an alternative measure on which to base potentially disrupting impacts on coastal communities and ecosystems. However, only a few of these semi-empirical applications had addressed the spurious inference that is likely to be drawn when one integrated process is regressed on another. Furthermore, it has been shown that spurious effects are not eliminated by stationary processes when these possess strong long memory. Our results indicate that both global temperature and sea level indeed present the characteristics of long memory processes. Nevertheless, we find that these variables are fractionally cointegrated when sea-ice extent is incorporated as an instrument for temperature that in our estimations has a statistically significant positive impact on global sea level.

Keywords: Sea level, Temperature, long memory, fractional cointegration, semi-empirical methods

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1 Introduction

Coastal erosion, loss of coastal wetlands and increased risks of flooding are some of the negative impacts that rises in the sea-level would have on coastal communities and ecosystems [9]. Although the economic costs derived from some of these impacts have been found to be relatively small in terms of GDP losses [2], it is nevertheless important to improve the confidence on the estimates of the relationship between global average temperature and global sea level. Current physics based-models have been shown to be able only to partially replicate recent sea level rise observations based on global average temperature and semi-empirical approaches have been implemented in hopes of filling the predictive gaps in current physics-based models [11]. However, most of these studies have not considered the potential spurious inference that is likely to be drawn from a regression of two series with strong temporal properties such as global sea level and global temperature [12]. The cointegration analysis in Schmith et al. [13] corrects for spurious effects. However, while the authors find a statistically significant impact of sea level on temperature, they do not find statistical significance for the converse causal relationship which has important implications for climate change adaptation policies. Our study extends the work of Schmith et al. in two ways. First, we obtain fractional orders of integration for the series and present evidence of a a long-run relationship between sea level and temperature that is fractionally cointegrated. Second, through the use of instrumental variables we obtain a consistent estimate of the impact of global average temperature on global sea level.

When the distant past of a series affects its current levels, it is said that the process possess long memory. Granger and Joyeaux [6] formally introduced the related concepts of long memory and fractional differentiation into the field of econometrics. Unit root tests are frequently used to detect the non-stationarity of a series, however, such tests are ill-suited in determining whether a series presents long memory. Tsay and Chung [15] showed that it is the presence of long memory even in stationary series what leads to spurious relationships. Therefore, determining the fractional order of integration is crucial if valid inferences are to be made regarding the statistical relationship between two time series.

Although the approach taken in [13] has been long recognized as an appropriate mechanism to correct for the non-stationary of the series in a regression context, the study does not incorporate more recent developments regarding the implications of long memory in series that seem to be stationary in the usual integer context. As mentioned above, our study extends the work of Schmith et al. to the fractionally integrated case allowing us to make statements about the long memory properties of the series and their implications in terms of spurious statistical inference.

Based on our estimates both global temperature and sea level indeed present the characteristics of long memory processes. We however find evidence that supports the fractional cointegration of these two series when sea-ice extent is incorporated as an instrument for sea level to explain global temperatures. The purpose of including an instrument for temperature is the inconsistency that would result from estimating ordinary least squares due to the simultaneity between sea level and temperature, since as empirically shown in [13], the former also affects the latter. Sea-ice extent is believed to be a valid instrument since it does not affect sea-level but it is affected by global temperatures. Our results show that global average temperature has a positive impact on global sea level.

The remainder of this paper is organized as follows. Data and methodology are respectively described in sections 2 and 3. Results are reported in section 4 and section 5 concludes.

2 Data

Annual observations on average global sea level and temperature anomalies for the period 1880-2009 were respectively obtained from [4] and [7]. Except for the slightly longer period considered in our estimations, these are practically the same data used in [11] and [13]. These two series are depicted in figures 1 and 2. They both show a clear upward trend during these period.

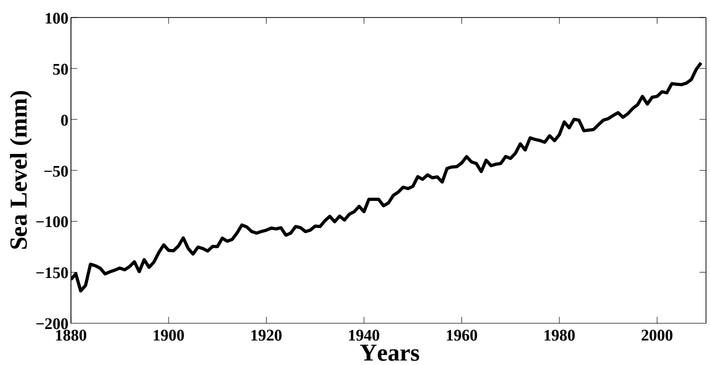

Figure 1: Sea level (1880-2009)

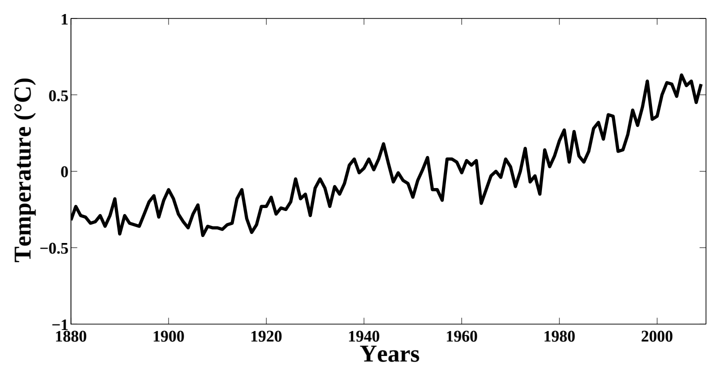

Figure 2: Global temperature (1880-2009)

Data on the mean annual sea-ice extent in the northern hemisphere (km^2) from 1880 to 2009 were obtained from Chapman,[1].Figure 3 shows the downward trend in the series.

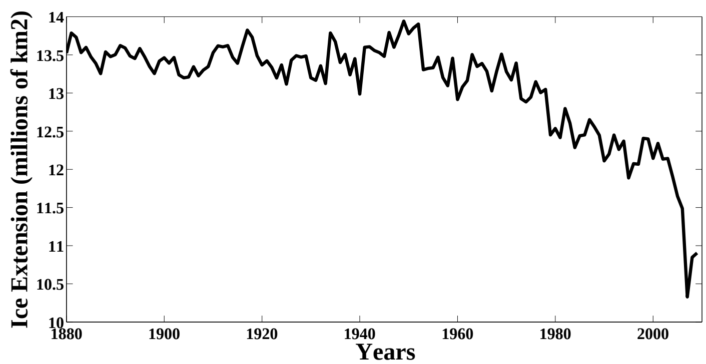

Figure 3: (a) Sea-ice extent 1880-2009

Figure 4 shows the correlograms for the three series up to the 100th lag; these patterns suggest the presence of long memory in the three series.

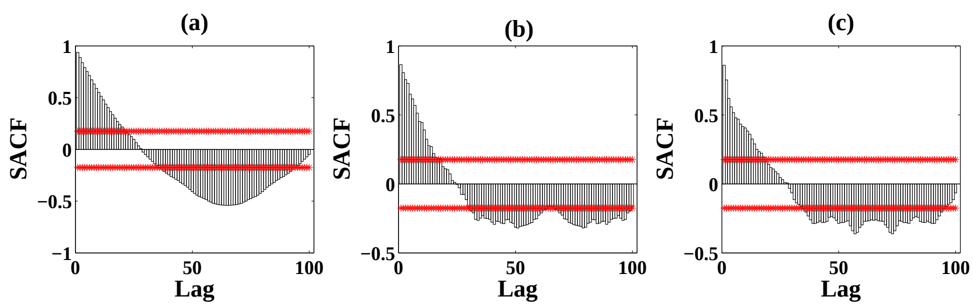

Figure 4: **(a) sea-level; (b) temperature; (c) sea-ice extent**

The results from statistical tests on the long memory of the three series are presented in section 4 after a description of the method in the following section.

3 Method

Long memory processes represent a bridge between those presenting infinite (non-stationary) and short memory (stationary). The autocorrelation function of processes of the latter kind exhibit exponential decay. Conversely, the autocorrelation function

[1] `http://arctic.atmos.uiuc.edu/SEAICE`. We would like to thank Prof. William Chapman from the University of Illinois for making available sea-ice extent data for our period of study.

in integrated processes does not decay, whereas the autocorrelation function of long memory processes decays hyperbolically.

The most common long memory processes are the autoregressive fractionally integrated moving average in which the order of integration includes the interval $-\frac{1}{2} < d < 1$, excluding the zero. These processes admit an infinite moving average or an infinite autoregressive process ([6]; [8];[14]).

We use the local Whittle estimator in order to estimate the fractional order of integration of the series [1]. Series are said to be stationary if $\frac{1}{2} < d$ and non-stationary otherwise.[2] Estimates of a regression between variables that are non-stationary may be spuriously significant. However, there may exist a linear combination that reduces their order of integration measured in the residuals of the estimated linear regression. In such cases the residuals become stationary and the series are said to be fractionally cointegrated and purged of spurious effects ([5]; [3]).

4 Results

In order to obtain formal evidence of long memory or non-stationarity of the series, the Whittle local statistic is estimated for both series [1]. Results are presented in Table 1 for different lag values (m) specified for the Whittle estimator. When $0 < d < 1$ the series are characterized by long-memory; furthermore if $0 < d < \frac{1}{2}$ the series would be stationary and if $\frac{1}{2} < d < 1$ they would be non-stationary. Based on the results from the Whittle local statistic, temperature, sea level and sea-ice extent are all non-stationary and present long memory.

Table 1: Results from the Whittle local statistic

Variable	$\mathbf{m = \frac{N}{4}}$	$\mathbf{m = 34}$	**Default**
Temperature	d=0.816	d=0.841	d=0.9031
	(0.088)	(0.085)	(0.121)
Sea level	d=0.938	d=0.944	d=0.929
	(0.088)	(0.085)	(0.121)
Ice extent	d=0.726	d=0.755	d=0.612
	(0.088)	(0.085)	(0.121)

Correlations between sea level S and temperature T are shown in table 2 for the original variables in levels and filtered series based on the $m_1 = \frac{N}{4}$ and $m_2 = N^{0.6}$.[3]

[2] When $d < 0$, the series are said to be antipersistent.

[3] m_2 is the default selection of lags for the whittle estimator in the software GRETL, which in this case is 17.

Table 2: Correlation between global mean temperature and sea level

	Coefficient	**p-value**
$\rho_{M,T}$	0.897	< 0.001
$\rho_{\Delta^{0.928}M,\Delta^{0.903}T}$	0.278	0.0013
$\rho_{\Delta^{0.938}M,\Delta^{0.815}T}$	0.292	< 0.001
$\rho_{\Delta^{0.944}M,\Delta^{0.841}T}$	0.288	< 0.001

The correlation between the original series (0.897) is similar to that found in [11] (0.88). However, as noted in [12] these correlation does not take into account the potential trend in the series that could result in a spurious relationship. The corrected correlations obtained with the filtered series are much weaker than that between the original non-stationary series.

Our results reported in table 3 are based on an instrumental variables regression in which two lags of the sea-ice extent series serve as instruments for temperature. We implement this method in order to overcome the potential simultaneity between sea-level and temperature. Past levels of sea-ice extent could be considered valid instruments since they should not be correlated with sea level except through their indirect impact on temperature. Tests on the relevance and validity of sea-ice extent as an instrument for temperature are also reported in table 3.

Table 3: IV Regression (dependent variable: sea level)

Variable	**Coefficient**	**Standard error**	**z-statistic**	**p-value**
Constant	-61.308	3.592	-17.069	0.000
Temperature	218.831	13.578	16.116	0.000
R^2		0.806		
Sargan OID-test (p-value)		0.809		
Weak instruments F-test (p-value)		0.000		
Observations		128		

Heteroscedastic-autocorrelation robust standard errors.

Based on the Whittle estimator, the residuals from the regression in table 3 have an order of integration of 0.4114 which suggests that the series are fractionally cointegrated. Importantly, the instruments are valid and relevant according to the Sargan over-ideintification (OID) and the weak-instruments F-test. Our estimations indicate that temperature has a positive and statistically significant impact on sea level: for each °C increase, sea level would rise by 21.8 cm. In the absence of temperature increases, sea level would be reduced by 6.1 cm. The magnitude of these impacts are within the range found in [11] who estimate a sea level rise of 125 cm for an increase of 5.8 °C (i.e., 21.6 cm per °C). As in [11] our estimates are above those found for the six emissions scenarios in the IPCC report [10] that range from 9.2 to 16.4 cm per °C. It is important to note that our estimates can not incorporate future non-linear changes in the relationship between temperature and sea level that are likely to occur due to melting of land-based ice at rates that have not been observed in our sample period.

5 Conclusions

This study provides a consistent estimate of the impact of temperature on sea level. Our results suggest that sea level would rise by roughly 22 cm per °C increase. This estimate together with the global temperature increases from IPCC's six emissions scenarios would result in a global sea level rise between 24 and 141 cm by the end of the 21st century. However, as with other semi-empirical estimates, our predictions are not capable of incorporating potentially non-linear effects deriving from land-based ice melting. Therefore, our estimates should be taken as a lower-bound impact of temperature on sea level rises that will ultimately inflict severe damages on coastal communities and ecosystems.

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