Stationary long memory or nonstationary series with stationary increments?

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Abstract

A few proxy-based air temperature series being previously analyzed assuming stationarity and long memory are reanalyzed. The traditional approach modeling by means of autoregressive and integrated moving average models is applied. The latter method produces more determined representation of the series variability.

1 Introduction

Modeling climate time series appeared to be a useful tool to understand climate variability. Examination of dependence between the time series terms is crucial for finding an applicable model. Several studies have shown that this dependence may contain long memory (e.g. Hurst 1951). Recent studies have shown that several different methods can be applied to study processes with expectable long memory (e.g. Mills 2007, Kärner and McKitrick 2009).

In mathematics a stochastic (random) process $\xi(t)$, $-\infty \leq t \leq \infty$ is a family of real-valued random variables corresponding to observations made at a regular time intervals t. At each moment the variable $\xi(t)$ has a certain probability density. The probability structure describing the relationship between the variable at different times is an important characteristic of stochastic processes (e.g. Box et al 1994).

The term *time series* is used to denote sample realizations $X(t)$ where $t = 1, 2, \dots, n$, of the stochastic processes. The actual length of the interval between the successive terms (from day to century in climate series) is crucial for physical interpretation of the temporal variability in time series but, generally does not change the mathematical approach necessary to study it.

Quantitative characterization of stochastic processes needs to encompass both stationarity and nonstationarity. A stochastic process is *weakly* (also *covariance*, or *wide-sense* stationary if its mean value is constant and its autocovariance function is invariant under translation. In data analysis the term is generally used in its weak sense.

1.1 Two interpretations of the stationarity definition

In practical applications of stationary random processes the covariance function $C(\tau)$ usually tends to zero as lag τ grows sufficiently (Khinchin (1934), Yaglom (1986a) p. 103). This condition introduces a customary stationarity treatment accepting the situation where a finite correlation length is naturally connected to the stationarity. Yaglom (1986a pages 115-140) keeps that discrimination level studying stationary processes. His analysis comprises autocorrelation functions which contain an exponentially decreasing factor.

The same philosophy is applied in fitting autoregressive moving average (ARMA) or autoregressive integrated moving average (ARIMA) models to empirical time series (Box et al. 1994). The primary tool for model identification in that theory is the sample autocorrelation function. If its decay is sufficiently fast the stationary ARMA family models are suitable for representing the series' temporal variability. In the case of a slow decay of the autocorrelations, a use of nonstationary ARIMA models is suggested. Such an approach to select the model is strongly influenced by a principle of parsimony by Tukey (1961). It states that in a model selection it is important to employ the smallest possible number of parameters for an adequate representation (Box et al. 1994 p 16). Thus, it is logical that Parzen (1986) introduces *a framework to integrate the analysis of long memory (nonstationary) time series with the analysis of short memory (stationary) time series*.

The modern time series analysis prefers to follow strictly the primary definition of stationarity. and defines stationary long memory processes (e.g. Beran (1994):

> Let $\xi(t)$ be a stationary process for which the following holds. There exists a real number $0 < \alpha < 1$ and a constant $c_\rho > 0$ such that
> $$lim_{k \to \infty} \rho(k)/[c_\rho k^{-\alpha}] = 1, \qquad (1)$$

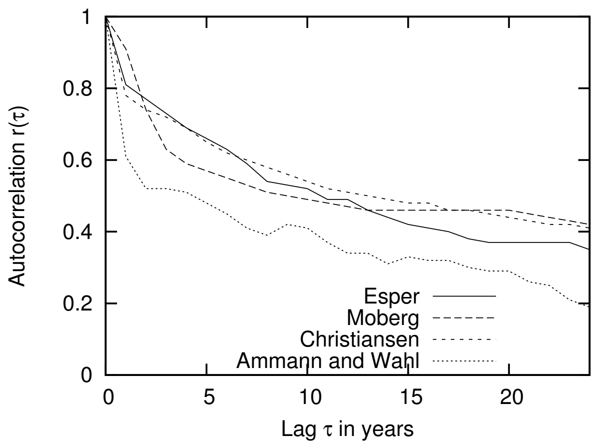

Figure 1: Sample autocorrelations for air temperature time series by Esper et al. (2002), Moberg et al.(2005), Ammann and Wahl (2007), and Christiansen and Ljungqvist (2012).

where $\rho(k)$, $k = 0, 1, 2, ...$ stands for its correlation function. Then $\xi(t)$ is called a stationary process with long memory or long range dependence.

It is important to study the influence of that determination difference to time series modeling. Analysis of long range air temperature series is of primary interest here.

1.2 Treated time series

Four yearly time series are studied in this example. The series have been recently constructed with different methods from a variety of proxy data by Esper et al. (2002), Moberg et al. (2005), Ammann and Wahl (2007) and Christiansen and Ljungqvist (2012). Three of them contain the Northern Hemisphere near surface air temperature and the series by Christiansen and Ljungqvist (2012) contains the temperature over the belt from 30 to 90 degrees NH.

It is useful to illustrate the decay rate for the corresponding sample autocorrelations. Figure 1 shows it up to 24 year lag.

All four series have slowly decaying autocorrelations. Their behavior at least approximately satisfies the long memory criterion Eq. (1) Next the difference between two stationarity interpretations will be examined using the function designed to describe the variability of series with stationary increments.

Different approaches of analysis may arise from different interpretation of the slow decay of correlation shown in Figure 1

1. Mills (2007) studied the variability of Moberg et al. (2005) series more carefully. He computed its sample autocorrelation function up to 200 lags. The persistence and slow decay of the sample autocorrelation function, where all the first 200 autocorrelations are significantly positive provided further confirmation of existing long memory. As a result he fitted an autoregressive fractionally integrated moving average (ARFIMA) model to describe the variability. After some tests he still did not fully excluded a possibility that NH temperatures by Moberg et al. (2005) might be nonstationary.

2. Totally different way of modeling can be obtained using a scheme developed by Box et al. (1994). This philosophy suggests that the slowly decaying autocorrelation is an indication that there may exist an appropriate model for the increment series.. An exploitation of that idea will be described later on.

To study the influence of these ways upon the modeling results it is reasonable to apply the theory of processes with stationary increments.

2 Stochastic processes with stationary increments

A wide class of stochastic processes those properties exclude stationarity are customarily called nonstationary processes. Important sub-class of the nonstationary processes are those with stationary first order increments. Lovejoy and Schertzer (1984) showed that many long range air temperature series belong to this sub-class.

A stochastic process $X(t)$ is said to have stationary increments if the distribution of $X(t+\tau)-X(t)$ is independent of t for every τ. Because every stationary process is also a process with stationary increments (e.g. Yaglom 1986) a method of analysis capable of examining properly series with stationary increments is preferable. Temporal variability in a stationary series is characterized by means of the autocorrelation function. The characterization of series with stationary increments reduces to computation of the second moment of the series (1-st order) increment as a function of the increment interval τ (Yaglom 1986):

$$D(\tau) = E[(X(t+\tau) - X(t))^2]. \quad (2)$$

If $X(t)$ is also stationary (i.e. the covariance depends upon τ only) one can open the brackets and produce (Yaglom 1986 p 395)

$$D(\tau) = 2[C(0) - C(\tau)], \quad (3)$$

where $C(\tau)$ stands for autocovariance. Provided that the autocoariance vanishes for large τ, Eq (3) shows that this event coincides with the saturation of $D(\tau)$ at the level of twofold variance. This means that in the case of long term stationarity one can separate two scales for the series. The first corresponds to the growth of $D(\tau)$ and the other to saturation. The first one could be regarded as the scale of initial (short range) nonstationarity.

The growth of $D(\tau)$ does not always reaches saturation. Another important feature is scaling. The term *scaling* (or *scale invariance* is often (e.g. Barenblatt 2003) used to denote a power law relationship between the function and its argument. Scaling of the structure function

$$D(\tau) = A\tau^{2H}, \quad (4)$$

where $A > 0$ and $0 < H < 1$, plays important role in data analysis.

The stochastic processes whose structure functions have the scaling property (4), are called self-similar (Yaglom 1986, p. 408) The processes with stationary increments which have scaling structure function of the form (4) were first studied by Kolmogorov (1940). The normal (Gaussian) processes with stationary increments with scaling structure function are often called fractional Brownian Motions (FBM), since they can be obtained from an ordinary Brownian motion by means of a fractional derivation.

The exponent value determines different statistical properties for FBM's. If $0 < H < 0.5$ the processes are called *antipersistent* because the correlation between consecutive increments of the process remains negative regardless of the increment interval. If $0.5 < H < 1.0$ the processes are called *persistent* because the correlation between consecutive increments of the process remains positive regardless of the increment interval (Mandelbrot and van Ness 1968). This is one introduction of the term persistence in time series analysis, connected to the processes with stationary increments. Another use of the term will be introduced later on in connection with stationary processes.

2.1 A spectral criterion for stationarity

Self-similarity relationship in spectral domain is written as

$$p(\nu) \propto \nu^{-\beta}, \quad (5)$$

where $p(\nu)$ is the power spectrum, ν is the frequency and β is the scaling exponent. (e.g. Mandelbrot 1982).

On the basis of the Wiener-Khinchine theorem there exists the relationship

$$\beta = 2H + 1, \qquad where 0 < H < 1. \quad (6)$$

between the scaling exponents β and H for these two functions $p(\nu)$ and $D(\tau)$, respectively. Mandelbrot (1982) found that the criterion $\beta < 1$ is necessary for stationarity of self-similar processes.

Eq. (6) enables one to suggest the corresponding criterion ($H = 0$) applicable for stationarity for arbitrary (i.e. not self-similar) series with stationary increments. It is possible that there exists some nonzero increment interval before the saturation when $D(\tau)$ is growing. This means that on the basis of H there are two different (increment) regions of temporal variability. The question remains how to interpret this increment range before the saturation. It is simple to treat that τ range as a region of nonstationarity. (e.g Davis et al. 1994 Kärner 2009).

Computing the structure function growth for four earlier selected proxy based temperature series enables us to get more detailed picture about the series variability. Figure 2 shows a log/log plot of the growth of $D(\tau)$ computed from the same four yearly temperature series up to the increment range 256 years ($\log_2 = 8$).

Figure 2 shows that the growth of $D(\tau)$ for four original temperature series does not saturate. This means that the series should be further analyzed as being nonstationary with stationary increments.

1. Three of the series show good monoscaling over the interval from 1 to 256 years. The corresponding exponent H estimates in Eq 4 are .16, .14, and .08 (the latter for Ammann and Wahl).

2. The series by Moberg et al (2005) shows more elaborate variability. It can be treated as biscaling because its variability for the available short increment range (up to 4 years) has remarkably higher intensity (in terms of H) than over the remaining increment range. Approximating H over the τ region from 0 to 8 gives $H = .16$, but over the interval from 2 to 8 $H = .09$.

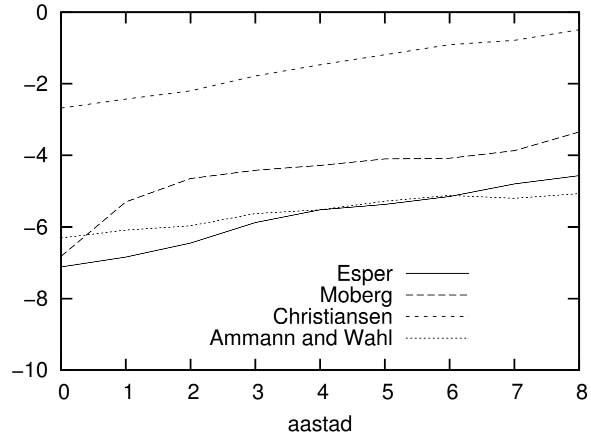

Figure 2: Log/log plot showing the growth rate of $D(\tau)$ over the increment range from 1 to 256 years

3. As a result, all four series behave like antipersistent nonstationary series with stationary increments over the increment range 1 to 256 years.

Different growth rate of $D(\tau)$ for original series generally means that certain conditions to find an applicable model will change together with the scale break for $D(tau)$ (see Kärner 2009 for details how to get an acceptable model to represent long range variability in daily total solar irradiance series).

3 The $D(\tau$ growth for accumulated series

If the growth rate of $D(\tau)$ for series $X(t)$ saturates, then the corresponding scaling exponent reaches zero and terminates the quantifying of a series with stationary increments by means of it (i.e. H).. In this case the both (necessary) criteria for stationarity of the series $X(t)$ are satisfied. I.e. that for the time domain ($D(\tau)$) gives $H = 0$ and that for the spectral domain gives $\beta = 1$ (see Eq. 6). As long as in the spectral domain the value $\beta = 0$ (white noise) also corresponds to a stationary series, there appears a stationarity interval $0 \leq \beta \leq 1$ in terms of β for that series. In the case of stationary series $X(t), t = 1, \ldots, n$ the series

$$y(t) = \sum_{i=1}^{t} X(i) \tag{7}$$

from any available start value for $i = 1$ appears to be a sample from a process with stationary increments and we can use $D(\tau)$ to quantify it. It is possible that the accumulated series satisfies the scaling relationship (4) for $D(\tau)$ with exponent H_a[1].

In this case, three classes can be separated:

1. The situation $0 < H_a < 0.5$ has been found to be very exceptional in actual data analysis (Beran 1994).
2. In the case of $H_a = 0.5$ no long memory is noticed.
3. In the case of $0.5 < H_a < 1$ the long memory is present in the original $X(t)$ series.

Studies about the scaling of accumulated series is often used to detect long memory in time series (e.g Talkner and Weber 2000, Eichner et al. 2003) The following example is assumed to show that the real detection is a nontrivial problem. Due to high sensitivity of $D(\tau)$ to autocorrelation of the original series it might be difficult to distinguish between long memory and any mixture of all range memories.

Figure 3 contains two panels to illustrate growth of $D(\tau)$ for two weakly correlated stationary series $X(t)$ and their accumulated counterparts $y(t)$. To help comparison with the scaling proportional to $\tau^0.5$ the corresponding line is shown in both panels. On the upper panel the initial series is generated by means of the first order autoregressive model $X(t) = AX(t-1) + a(t)$, where A=0.3 and $a(t)$ is white noise. In the lower panel $X(t)$ stands for daily precipitation amount series at Mont Aigoual (France) from 1946 to 2008, downloaded from `eca.knmi.nl` (see Klein Tank et al. 2002 for details). The daily amounts are weakly correlated having only the first two coefficients nonzero ($r(1) = -.30$, $r(2) = -.15$).

[1]Hereafter, to avoid confusion the exponent for accumulated series is marked by H_a

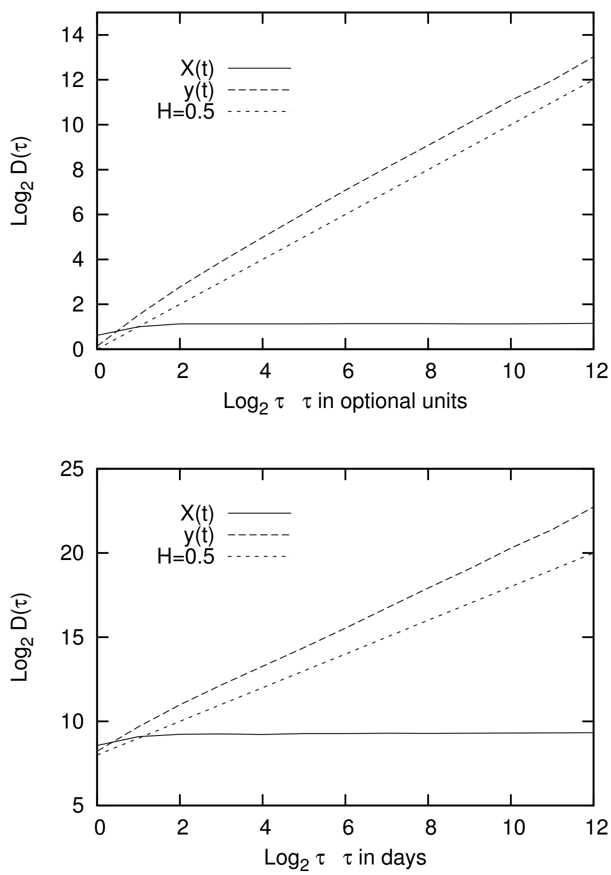

Figure 3: Log/log plot showing the growth rate of $D(\tau)$ for two weakly correlated series $X(t)$ and its accumulated counterpart $y(t)$. Upper panel: $X(t)$ stands for simple AR(1) variable with the coefficient 0.3. Lower panel: $X(t)$ stands for the precipitation amounts at Mont Aigoual (France) The growth rate proportional to τ is also shown in both panels

Figure 3 shows that $D(\tau)$ saturates very fast for both $X(t)$ series. The influence of correlation is hardly noticeable. Their behavior is totally different from those for air temperature series shown in Figure 2. But $D(\tau$ growth for both $y(t)$ series differs essentially. On the upper panel in corresponds precisely the comparison line as $log_2\tau$ grows beyond 2. But on the lower panel it grows evidently faster which means that the corresponding $H_a > 0.5$. This is an evident signal of long memory. It must be noted that the detection has been carried out following the interpretation by Yaglom (1986).

Comparison of figures 3 and 2 reveals the crucial role of the correlation between $X(t)$ terms for the scaling exponent Ha value.

3.1 Example on modern long memory detection

Temporal variability in the series by Esper et al. (2002) and by Moberg et al. (2005) has been carefully studied by Rybski et al (2006) using the method initiated by Hurst (1951). They started from sample autocorrelations of the series (see Figure 1 as an example including also two series analyzed by Rybski et al 2006). They defined the term *persistence* close to that by Beran (1994) (see Eq 1)

> " The term long-term persistence refers to auto-correlation functions which decay by a power law and are characterized by an infinite correlation time. Due to the long-term correlations, the variability on long time scales is strongly enhanced.[2]

Main goal of the paper by Rybski et al. (2006) was to show that all 6 records they analyzed are characterized by pronounced long-term persistence. To do that, they analyzed accumulated series $y(t)$ (see Eq 7) where $X(t)$ stand for the reconstructed temperature records. Analysis of the scaling properties of the accumulated series became popular after the paper by Hurst (1951).

But many researchers do not perceive the peculiarity of the Hurst's work in comparison with the customary analysis of time series variability. Hurst (1951) examined the accumulated series of Nile river level height because his goal was to study the water accumulation peculiarities in hypothetical reservoir (see Feder 1988 for detailed description of the problem). He was not interested in pure level variability at all. Generally, the integral of a function has different properties than the function under the integral mark (excluding e^x). The variability of $y(t)$ has been studied in several papers (e.g Talkner and Weber 2000, Eichner et al. 2003 and references therein) but the possibility of that variability to model the initial series $X(t)$ remained obscure.

Rybski et al. (2006) concluded:

> The previous claim that the most recent warming, observed by quality controlled instrumental data, would be inconsistent with the hypothesis of purely natural dynamics [list of papers] is supported by our long-term persistence analysis of different proxy-based reconstructions extending over many centuries and even up to two millenia.

This conclusion has no relation to the variability of original proxy-based temperature series $X(t)$ because the authors were unable to extract any useful property for $X(t)$ from the obtained scaling for $y(t)$. Citation the hypothesis about purely natural dynamics is improper because Rybski et al (2006) studied only climate response series. They had no idea about possible variations of the main forcing - i.e. total solar irradiance during the last two millenia.

3.2 Traditional interpretation using ARIMA models

Traditional interpretation of slowly decreasing autocorrelation function is well known for these statistical modelers who are used to apply ARIMA family models. Figure 2 shows that the structure function describes a certain scaling (i.e. $H > 0$) not saturation as the increment interval τ increases. This means that an examination of the behavior of $y(t)$ (Eq 7) to reveal the variability of $X(t)$ is inappropriate in the current situation. The obtained monoscaling with low H shows that an ARMA model for the series $X(t)$ increments $X(t) - X(t-1)$ should be applicable. The statement is confirmed by the autocorrelations of the yearly increment series $x(t) = X(t) - X(t-1)$, shown in Figure 4

Figure 4 shows important features of the increment series.

1. Behavior for three correlations appears to be very similar. Only $r(1)$ for each three differs evidently from zero. The others are much closer to zero. $r(2) = -.12$ for the increment series from Ammann and Wahl (2007) appears to be the next. Following the instructions by Box et al. (1994), the only non-zero correlation $r(1)$ suggests that the first order moving average MA(1) model might be appropriate to describe the increment series temporal variability. The model can be written as

$$X(t) - X(t-1) = a(t) - \Theta a(t-1), \tag{8}$$

[2] This is another popular use of the term *persistence* in time series analysis. Although it holds for stationary series (the previous one deals with the series with stationary increments) it may easily to cause certain confusion among the researchers.

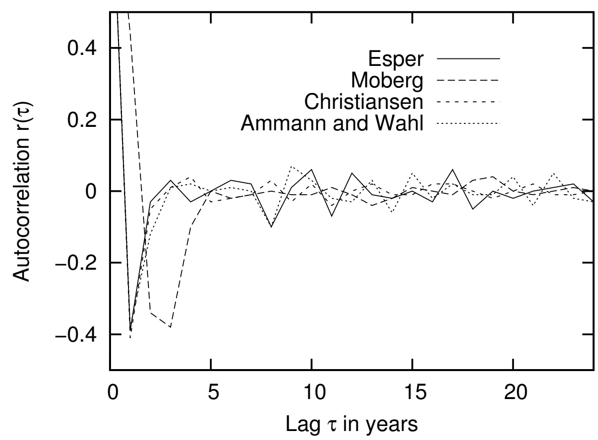

Figure 4: Sample autocorrelations for the yearly temperature increment series by Esper et al. (2002), Moberg et al. (2005), Ammann and Wahl (2007), and Christiansen and Ljungqvist (2012).

Table 1: .

Series	area	years	n	Λ	Q	99% critical
Esper	NH	831 - 1982 AD	1162	.486	39.6	41.6
Christiansen	30-90 NH	1 - 1973 AD	1973	.413	40.3	41.6
Ammann&Wahl	NH	1000 - 1980 AD	981	.285	47.7	41.6
GLIMPSE	Global	1600 - 1999 AD	400	.659	29.9	41.6

where Θ is the fitted coefficient and $a(t)$ is white noise. To describe the temperature $X(t)$ series, the model takes the form;

$$X(t) = \Lambda \sum_{i=1}^{\infty} a(t-i) + a(t), \tag{9}$$

where $\Lambda = 1 - \Theta$.

Fitting information for three proxy-based series together with one earlier example are shown in Table 1. A model generated series (named GLIMPSE) of global annual mean temperatures is modeled at `thejll.com/ESF.shtml`. The series contains results of a simulation of the climate of the last four centuries with a state-of-the-art coupled atmosphere-ocean general circulation model by Stendel et al (2006). Three last columns show important fitting results, the fitted coefficient $0 < \Lambda < 1$, the obtained portmanteau statistic Q and its 99% critical level at the used degree of freedom. One column shows that one version from the three did not pass the portmanteau test at that level.

2. The increments for Moberg et al. (2005) series lead to more mazy correlations. In addition to the approach used by Mills (2007) another model can be applied if accepting two different τ scales in the temporal variability and focusing on the longer range variability only. Such an approach is familiar from modeling various daily total solar irradiance and air temperature series (see Kärner 2009 for details). Figure 2 shows that for 4 year and longer increments the growth rate of $D(\tau)$ stabilizes to be approximately the same rate as for three other series. Low and stable growth rate enables us to apply ARIMA philosophy to fit this type of model over somewhat longer time step than the original yearly time step. Such an attempt leads to the same IMA(0,1,1) model but applied over 4 year time step. In this case 4 models are fitted over different sub-series separated from the original time series over 4 year interval each. The corresponding parameter Λ values are .230, .196, .242, .302, respectively. This gives the mean IMA(0,1,1) model with $\Lambda = .242$. The portmanteau statistic was remarkably less than the critical one at 99% significance level for all 4 cases.

   Mills (2007) asked the question whether the temperature series by Moberg et al. (2005) appears to be a sample from stationary long memory or nonstationary process? The answer via IMA(0,1,1) model is clear: nonstationary with stationary first order increments.

The obtained IMA(0,1,1) model has been applicable earlier to represent the temporal variability of daily total solar irradiance (TSI) at the top of the atmosphere (TOA) and several station based surface air temperature series (Kärner 2009, Kärner and de Freitas 2011).

Since the IMA(0,1,1) process is nonstationary, it does not vary in a stable manner about a fixed mean. But the exponentially weighted moving average can be regarded as measuring the *level* of the process at time t (see Box et al 1994 p 113). This shows that each new level is arrived at by interpolating between the new observation and the previous level. The level change depends on Λ. If it is small new level would rely heavily on the previous level. Because the level changes at every step by a random increment, the producible trend will be random. This means that the traditional trend calculation is unreliable to describe the temporal variability of the series produced by an IMA(0,1,1) process. More reliable knowledge could be obtained by means of Λ. Table 1 shows that the global yearly mean series models may have widely varying Λ's. This is probably due to different schemes and proxies used to reveal the temperature.

4 Concluding discussion

Samorodnitsky (2006) writes that the stationary long memory processes form the layer separating the nonstationary processes from the *usual* stationary processes. Applying the theory of stochastic processes with stationary increments enables us to estimate whether (and in what conditions) this layer really exists for time series analysis.

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